# Physics IA: How does the temperature affect the viscosity of glycerol?

## 1 Introduction

Living in the North required its own precautions: I wore minimum 3 layers of clothing every day when commuting to school to protect against the cold in the mountainous areas. However, I have always wondered, what dangers does the cold really pose in our body? I have always noticed that my heart beats faster in order to keep my body warm. Indeed, the function of the heart is to transport blood around the body, however, what would happen if this blood got too cold? Quick research leads me a link to the viscosity of the blood causing a clot within veins, causing direct health concerns and facts such as that for each 1.8°F or 1°C reduction in temperature on a single day is associated with around 200 additional heart attacks; that there are $53\%$ more heart attacks in winter; and the highest cold-induced cardiovascular risk exists just hours/days after exposure to cold (Omega, 2016).

Unfortunately finding the effects of cold on blood in veins requires an immeasurable amount of equipment and variables to count, therefore for convenience, I have decided that I will investigate the effect of cold in one of the materials that are moderately present in blood: glycerol which effectively represents the oil in our blood. The dynamic viscosity of a fluid such as blood measures its resistance to flow and is measured in Pascals second ($Pa \cdot s$), otherwise $kgm^{-1}s^{-1}$ as we know that Pascal is pressure defined as force ($kgms^{-2}$) over area ($m^2$) $\implies 1Pa = kgm^{-1}s^{-2}$. As a result of this research, I do believe that a higher viscosity would be very dangerous to a human's well-being. This results in asking myself "How does the temperature affect the viscosity of glycerol?"

To measure the viscosity of glycerol I will be utilising Stokes' Law.

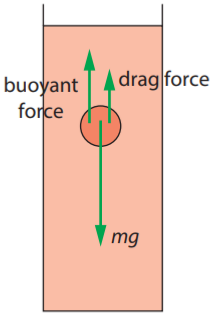


Figure 1: Sphere inside a fluid and its forces (Tsokos, 2014, p.44)

From Newton's second law we know that when a force is applied to a body of mass $m$ it accelerates. In this case, our acceleration is $g$ for our mass $m_s$ of the sphere. We will denote this as $F_g$. However, when the sphere touches the liquid, from Newton's third law, there is a force opposing the gravitation force of the sphere, the buoyant force $F_b$. This way, I can create a new equation for the net downward force $F_D$.

$$\begin{aligned} F_D &= F_g - F_b \\ &= m_s g - m_g g \end{aligned}$$

where $m_g$ is the mass of glycerol acting against the sphere. I will rewrite the masses using the fact that $mass = volume \times density$. I know that the volume of the sphere can be given as $V_s = \frac{4}{3}\pi r^3$. I will denote the densities of sphere and glycerol as $\rho_s$ and $\rho_g$ respectively. Since the buoyant force acts against the sphere, its volume can be written as the volume of the sphere as well. Therefore we can write the net downward force as

$$\begin{aligned} F_D &= V_s\rho_s g - V_s\rho_g g \qquad (1) \\ &= V_s g(\rho_s - \rho_g) \qquad (2) \end{aligned}$$

However, from this we can see that $\rho_s - \rho_g > 0$. This implies that $F_D > 0$. However, by Stokes' Law, there is another force called the dragging force $F_d$ that is proportional to its velocity acting against $F_D$. This force is given by

$$F_d = 6\pi\eta r v$$

therefore we get the final net force $F_{net}$ as

$$\begin{aligned} F_{net} =& F_D - F_d \\ =& F_D - 6\pi\eta r v \end{aligned}$$

For convenience I will let $k = 6\pi\eta r$. Therefore our equation for net force is

$$F_{net} = F_D - kv \tag{3}$$

This is a differential equation (since $\frac{dv}{dt} = a$) with a stable equilibrium point at $v = v_c$. When $v > v_c$, it will converge to $v_c$ due to the drag force's proportionality to velocity. That is, as the speed increases so does the drag force, but since drag force is described by speed and the force is proportional to acceleration from $F = ma$ acting in the opposite direction, converging to constant speed $v_c$. Similarly, when $v < v_c$, the drag force is less than the weight therefore it accelerates and converges to $v_c$. No matter the scenario, it will eventually reach a net force of 0 as it converges to the constant speed $v_c$ from Newton's first law. Using this, we can now solve for $\eta$ to obtain:

$$\begin{aligned} 0 &= V_s g(\rho_s - \rho_g) - 6\pi\eta r v_c \\ \eta &= \frac{V_s g(\rho_s - \rho_g)}{6\pi r v_c} \end{aligned}$$

Dimensionally we do indeed obtain $Pa \cdot s$

$$\begin{aligned} \eta &= \frac{m^3 \cdot ms^{-2} \cdot kgm^{-3}}{ms^{-1} \cdot m} & \implies \eta &= \frac{kgms^{-2}}{m^2 s^{-1}} \\ \implies \eta &= kgm^{-1}s^{-1} & \implies \eta &= Pa \cdot s \end{aligned}$$

Thus it is possible to see that $\eta$ is directly proportional to $\rho_s - \rho_g$ and inversely proportional to $v_c$.

## 2 Hypothesis

I believe that lower temperatures of glycerol will result in higher viscosity, and higher temperatures will result in lower viscosity.

As the temperature is defined to be the average kinetic energy of a particle in a material, the lower the temperature, the lower the average kinetic energy in a particle. As a result, less space is occupied by particles as they come closer together implying higher density. Consequently, a larger number for the density of glycerol $\rho_g$ (from a lower temperature) means that $\rho_b - \rho_g$ is a smaller number. Knowing that velocity $v_c$ is a constant, if we achieve a smaller number for $\rho_b - \rho_g$ in the numerator, the viscosity $\eta$ must be a smaller value due to its direct proportionality. Whilst the direct proportionality between the density difference and viscosity suggests the idea that the viscosity decreases, I do believe that there will be a substantial change in viscosity independent of the density which will outweigh the density change as the temperature varies. When I introduce the variable $T$ as temperature into the equation, I believe that the change in density will be minimal. If we consider equation 3 during terminal velocity:

$$0 = F_D - kv_c$$

Since my velocity is a constant, and since I hypothesise that the density difference is minimal, we can assume that $F_D$ is also a constant. This means that my temperature must affect $k$. However, $k = 6\pi\eta r$ and $r$ and $6\pi$ are constants, therefore temperature will directly affect $\eta$. The relationship between $T$ and $\eta$, however, is described to be "no comprehensive theory on the viscosity of liquids so far because of its complex nature." (Association, 1963), therefore there also does not exist an equation that perfectly describes the relationship of $\eta$ and $T$. However, I do believe that as the the temperature will increase, the viscosity will decrease which will also result in the decrease of the drag force. This means that there is less energy required to break the bonds between the ball and glycerol. This energy is provided by gravitational energy $mgh$, and since less energy is provided into the system to break these bonds, it is naturally transferred into kinetic energy, implying an increase in the constant velocity. Therefore, with this hypothesis, I must also observe an increase in my value of constant velocity.

# 3 Experiment

## 3.1 Variables

### 3.1.1 Independent Variable

The independent variable is the temperature of glycerol. The intervals that are measured are $0, 5, 10, 15, 20, 25, 30\ ^{\circ}C$.

### 3.1.2 Dependent Variable

The dependent variable is the measured viscosity of the liquid with its respective temperature.

### 3.1.3 Controlled Variables

1. **The method at which the ball is dropped** - in order to ensure that the ball does not gain any extra acceleration from the throw of my hand, the ball has to be "dropped" by letting gravity accelerate the ball from the same place.
2. **The mass and the size of the ball** - The volume of the mass of the ball cannot change as it would directly affect the velocity observed, thus this has to be kept constant.
3. **Volume of Glycerol** - The volume of the glycerol would also affect the velocity of the ball in the fluid as a result of pressure, hence this must also be kept constant at $250\text{ml}$
4. **Distance between measuring cylinder and camera** - In order to ensure that any possible parallax errors are systematic rather than random between each experiment if any, the distance between camera for observation and cylinder is kept the same.

## 3.2 Apparatus

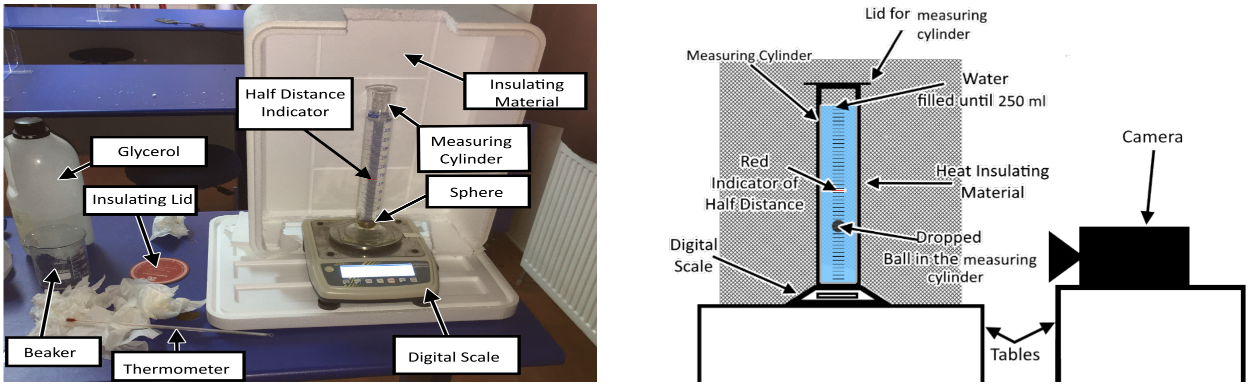
- Micrometer of uncertainty $\pm 5 \times 10^{-6}\text{m}$
- Digital scale of uncertainty $\pm 5 \times 10^{-6}\text{kg}$
- $\times 2$ long measuring cylinders of $250\text{ml}$ of uncertainty $\pm 1\text{ml}$
- $\times 2$ beakers
- Meter ruler of uncertainty $\pm 5 \times 10^{-4}\text{m}$
- Thermometer of uncertainty $\pm 0.05^{\circ}\text{C}$
- iPhone Camera
- Glycerol
- Styrofoam
- A sphere more dense than glycerol

## 3.3 Procedure

1. The beaker's height is measured using a meter ruler. This value is then halved and then marked in the beaker with a red marker. Furthermore, the beaker is also covered is surrounded with heat-insulating material in order to minimise energy transfer between surroundings and glycerol. The beaker is then placed on top of the digital scale.
2. A thermometer is placed inside a bowl filled with glycerol which is placed in a bath of water. This bath is then heated until the desired temperature. If the glycerol is overheated, it is then left at room temperature until it reaches desired temperatures.

3. For glycerol that requires lower temperatures, the glycerol is placed inside a refrigerator and cooled until required with a thermometer. If overcooled, the glycerol is then left at room temperature until it reaches desired temperatures. It is ensured that before measuring the temperature the glycerol is stirred for uniform distribution of heat.
4. Glycerol is then poured into the measuring cylinder until it reaches the $250\text{ml}$ mark. The beaker is then quickly covered with a lid to mitigate energy transfers and evaporation.
5. The weight is then recorded to calculate the density of the glycerol at that temperature.
6. The ball is then dropped lightly into the beaker and the falling is recorded using a camera.
7. The result is then analysed by recording the average velocity by measuring the time to reach the bottom of the beaker after the 'half-mark' indicated. This is to ensure that the ball is closer to the terminal velocity.
8. The glycerol is then re-used for further heating or cooling. This experiment is repeated for each temperature interval 2 more times, in order to obtain 3 sets of data to ensure higher accuracy of the data by then taking the average.

### 3.4 Setup



(a) Photo of the procedure

(b) Illustrated diagram of the setup

Figure 2: Diagram of the procedure using illustration and a photo

### 3.5 Environmental, Ethical and Safety Concerns

- No living beings were harmed in the making of this experiment
- No environmental concerns and harm was done in the making of this experiment
- Glycerol is poured carefully to also ensure that it does not splash into any equipment such as camera or surroundings, whilst ensuring conservation of the material for future use

### 3.6 Data results and Numerical Analysis

| | **Diameter**<br>$d_s/m$<br>$\Delta d_s = 5 \times 10^{-6}$ | **Mass**<br>$m_s/kg$<br>$\Delta m_s = 5 \times 10^{-6}$ | **Volume**<br>$V_s/m^3$<br>$\Delta V_s = 3 \times 10^{-9}$ | **Density**<br>$\rho_s/kgm^{-3}$<br>$\Delta \rho_s = 8$ |
|---|---|---|---|---|
| Sphere | 0.01946 | 0.03268 | $3.859 \times 10^{-6}$ | 8469 |

Table 1: Table of results of measured properties of the sphere

Answers were rounded off to 4 significant figures. The uncertainty of the density is rather small, thanks to the accuracy of apparatus used e.g. micro meter and high sensitivity digital scale. These uncertainties were calculated using:

$$\Delta V_s = V_s \left| 3\frac{\Delta d_s}{d_s} \right| \qquad \Delta\rho_s = \rho_s \left( \frac{\Delta m_s}{m_s} + \frac{\Delta V_s}{V_s} \right)$$

$$\Delta V_s = 3.859 \times 10^{-6} \left| 3\frac{5 \times 10^{-6}}{0.01946} \right| = 3 \times 10^{-9} \qquad \Delta\rho_s = 8469 \left( \frac{5 \times 10^{-6}}{0.03268} + \frac{3 \times 10^{-9}}{3.859 \times 10^{-6}} \right) = 8$$

| | **Mass of Glycerol + Measuring Cylinder + Lid** $m/kg$ $\Delta m = 5 \times 10^{-6}$ | | | | |
|---|---|---|---|---|---|
| **Temperature** $T/°C$ $\Delta T = 0.05$ | **Attempt I** | **Attempt II** | **Attempt III** | **Average Mass of Glycerol + Measuring Cylinder + Lid** $m_{avg}/kg$ **Measurement Uncertainty:** $\Delta m_{avg} = 5 \times 10^{-6}$ **Random Uncertainties are placed in each respective individual values.** | **Measuring Cylinder + Lid** $m_b/kg$ $\Delta m_b = 5 \times 10^{-6}$ |
| 0 | 0.56121 | 0.56218 | 0.56220 | $0.56186 \pm 5.0 \times 10^{-4}$ | 0.24585 |
| 5 | 0.56031 | 0.56163 | 0.56270 | $0.56155 \pm 1.2 \times 10^{-3}$ | 0.24585 |
| 10 | 0.56132 | 0.56030 | 0.55951 | $0.56034 \pm 9.1 \times 10^{-4}$ | 0.24585 |
| 15 | 0.52168 | 0.52310 | 0.52225 | $0.52234 \pm 7.1 \times 10^{-4}$ | 0.20848 |
| 20 | 0.52088 | 0.52282 | 0.52107 | $0.52159 \pm 9.7 \times 10^{-4}$ | 0.20848 |
| 25 | 0.52132 | 0.52201 | 0.52030 | $0.52121 \pm 8.6 \times 10^{-4}$ | 0.20848 |
| 30 | 0.52100 | 0.52154 | 0.51910 | $0.52055 \pm 1.2 \times 10^{-3}$ | 0.20848 |

Table 2: Table of results of measured properties of each measuring cylinder with Glycerol

Both of the measurement uncertainty and the random uncertainties were calculated. However, for future calculations, the random error will be used as is a significantly higher value. The random uncertainty was calculated by subtracting the smallest Attempt value from the largest, then dividing by 2. For the example calculation, data for $T = 0°C$ was used:

$$\Delta m_{avg} = \frac{0.56220 - 0.56121}{2} = 5.0 \times 10^{-4} \text{ to 1 s.f.}$$

| **Temperature** $T/°C$ $\Delta T = 0.05$ | **Average mass of Glycerol** $m_{g\ avg}/kg$ | **Volume of Glycerol** $V_g/m^3$ $\Delta V_g = 10^{-6}$ | **Density** $\rho_g/kgm^{-3}$ |
|---|---|---|---|
| 0 | $0.31601 \pm 5.0 \times 10^{-4}$ | 0.00025 | $1264 \pm 7$ |
| 5 | $0.31570 \pm 1.2 \times 10^{-3}$ | 0.00025 | $1263 \pm 10$ |
| 10 | $0.31453 \pm 9.1 \times 10^{-4}$ | 0.00025 | $1258 \pm 9$ |
| 15 | $0.31386 \pm 7.1 \times 10^{-4}$ | 0.00025 | $1255 \pm 8$ |
| 20 | $0.31311 \pm 9.7 \times 10^{-4}$ | 0.00025 | $1252 \pm 9$ |
| 25 | $0.31273 \pm 8.6 \times 10^{-4}$ | 0.00025 | $1251 \pm 8$ |
| 30 | $0.31207 \pm 1.2 \times 10^{-3}$ | 0.00025 | $1248 \pm 10$ |

Table 3: Density of glycerol calculations at different temperatures

Because the measurement uncertainty of the mass of the beaker is insignificant in comparison to the random uncertainty of the masses recorded for the mass of glycerol + beaker, the random error was directly carried on for future calculations. The average mass of glycerol was calculated by subtracting the beaker mass from the average mass. The uncertainty of the density was calculated by the formula below, and an for the following example calculation row 1 was used and answers kept to 1 s.f. for consistency.

$$\Delta\rho_g = \rho_g\left(\frac{\Delta V_g}{V_g} + \frac{\Delta m_{g\ avg}}{m_{g\ avg}}\right) \qquad \Delta\rho_g = 1264\left(\frac{10^{-6}}{0.00025} + \frac{5\times10^{-4}}{0.31601}\right) = 7 \text{ to 1 s.f.}$$

| | **Time from** $125$ **to** $0$ml<br>$t/s$ | | | | | |
|---|---|---|---|---|---|---|
| **Temperature** $T/°C$ $\Delta T = 0.05$ | **Attempt I** | **Attempt II** | **Attempt III** | **Average time** $t_{avg}/s$ | **Distance from** $125$ **to** $0$ml $d_b/m$ $\Delta d_b = 5\times10^{-4}m$ | **Speed** $s/ms^{-1}$ |
| 0 | 1.687 | 1.645 | 1.548 | $1.627 \pm 0.070$ | 0.1085 | $0.067 \pm 0.003$ |
| 5 | 0.783 | 0.733 | 0.723 | $0.746 \pm 0.030$ | 0.1085 | $0.145 \pm 0.007$ |
| 10 | 0.395 | 0.393 | 0.402 | $0.397 \pm 0.005$ | 0.1085 | $0.273 \pm 0.005$ |
| 15 | 0.391 | 0.386 | 0.378 | $0.385 \pm 0.007$ | 0.1085 | $0.282 \pm 0.006$ |
| 20 | 0.306 | 0.302 | 0.303 | $0.304 \pm 0.002$ | 0.1085 | $0.357 \pm 0.004$ |
| 25 | 0.219 | 0.237 | 0.238 | $0.231 \pm 0.010$ | 0.1085 | $0.470 \pm 0.022$ |
| 30 | 0.157 | 0.160 | 0.167 | $0.161 \pm 0.005$ | 0.1085 | $0.674 \pm 0.024$ |

Table 4: Speed of falling sphere calculations at different temperatures

The uncertainty of the time was calculated by taking finding the difference between the minimum and the maximum and dividing by two and rounding the answer off to 3 d.p.. Finding instrumental and measurement uncertainty of the camera proved to be difficult due to multiple possibilities and variables. Furthermore, it is likely that the random uncertainty is also greater, hence this decision should theoretically not create a big difference. A higher amount of trials would also enable me to be able to do the error using standard deviation, which would be more accurate.

Indeed, in lower temperatures a gradual time decrease can be seen for the ball to reach the bottom, expected as a result of absorption of heat from the surroundings. Similar effect can be seen within higher temperatures where the heat is lost to the surroundings instead, leading to decreased temperatures as time passes. This is somewhat suggestive of my hypothesis being true.

However, the difference in values between recorded low temperatures also follow an interesting pattern, given that the difference of speed between each temperature is more significant. Therefore, the effect of increasing and decreasing temperature on the viscosity may perhaps be more clearly seen when testing with lower temperatures but also lower intervals, as the gaps are observed to be larger than the rest. This would also, however, require more sensitive devices which I do not have a hold of.

Lastly, while a great difference of speed can be seen between each temperature interval, the speed of $0.282ms^{-1}$ for $T = 15°C$ seems to be an anomaly. The increase in speed between each interval is usually large, however, the speed difference for $T = 10°C$ and $T = 15°C$ is only $0.009$. This will be taken into consideration when doing future analysis and interpretation of results. The equations and example calculations for row 1 are below

$$\Delta s = s\left(\frac{\Delta t_{avg}}{t_{avg}} + \frac{\Delta d_b}{d_b}\right)$$

$$\Delta t_{avg} = \frac{1.687 - 1.548}{2} = 0.07 \qquad \Delta s = 0.067\left(\frac{0.07}{1.627} + \frac{5\times10^{-4}}{0.1085}\right) = 0.003 \text{ to 3 d.p.}$$

| **Temperature** $T/°C$ $\Delta T = 0.05$ | **Speed** $s/ms^{-1}$ | **Density Difference** $\rho_s - \rho_g/kgm^{-3}$ | **Radius of Sphere** $r/m$ $\Delta r = 2.5 \times 10^{-6}$ | **Volume of Sphere** $V_s/m^3$ $\Delta V_s = 3 \times 10^{-9}$ | **Viscosity** $\eta/Pa \cdot s$ |
|---|---|---|---|---|---|
| 0 | $0.067 \pm 0.003$ | $7205 \pm 15$ | 0.00973 | $3.859 \times 10^{-6}$ | $22.2 \pm 1.1$ |
| 5 | $0.145 \pm 0.007$ | $7206 \pm 18$ | 0.00973 | $3.859 \times 10^{-6}$ | $10.3 \pm 0.5$ |
| 10 | $0.273 \pm 0.005$ | $7211 \pm 17$ | 0.00973 | $3.859 \times 10^{-6}$ | $5.45 \pm 0.12$ |
| 15 | $0.282 \pm 0.006$ | $7214 \pm 16$ | 0.00973 | $3.859 \times 10^{-6}$ | $5.28 \pm 0.13$ |
| 20 | $0.357 \pm 0.004$ | $7217 \pm 17$ | 0.00973 | $3.859 \times 10^{-6}$ | $4.17 \pm 0.06$ |
| 25 | $0.470 \pm 0.022$ | $7218 \pm 16$ | 0.00973 | $3.859 \times 10^{-6}$ | $3.22 \pm 0.16$ |
| 30 | $0.674 \pm 0.024$ | $7221 \pm 18$ | 0.00973 | $3.859 \times 10^{-6}$ | $2.21 \pm 0.09$ |

Table 5: Calculation of glycerol viscosity at different temperatures

From an observation of the results, a negative exponential relationship can be seen, as the viscosity rapidly increases as the temperature decreases. The uncertainty of the viscosity was calculated by defining $\alpha = \rho_s - \rho_g$ and its uncertainty $\Delta\alpha$ for simplicity. Thus my uncertainty equations with example calculations from Row 1 are below:

$$\Delta\alpha = \Delta\rho_s + \Delta\rho_g \qquad \Delta\eta = \eta\left(\frac{\Delta s}{s} + \frac{\Delta r}{r} + \frac{\Delta V_s}{V_s} + \frac{\Delta\alpha}{\alpha}\right)$$

$$\Delta\alpha = 7 + 8 = 15 \qquad \Delta\eta = 22.2\left(\frac{0.003}{0.067} + \frac{2.5 \times 10^{-6}}{0.00973} + \frac{3 \times 10^{-9}}{3.859 \times 10^{-6}} + \frac{15}{7205}\right) = 1.1 \text{ to 2 s.f.}$$

### 3.7 Graphical Analysis

However, graphing our results is trickier than my initial thought. There is no comparable theoretical basis for the estimation of liquid viscosities. Thus, it is particularly desirable to determine liquid viscosities from experimental data when such data exists (Robert C. Reid, 1987, p.517). Many models exist to explain the behaviour of viscosity against temperature, however, for the purpose of this Internal Assessment, I will consider a widely used empirical equation using my own and literature discrete values. I then will compare these constants found between my values, literature discrete values and literature valaues of constants found by the same scientist who used this equation. The discrete literature values I will use for the analysis of separate equations are the following (Association, 1963)

| **Temperature** $T/°C$ | **Viscosity** $\eta/Pa \cdot s$ |
|---|---|
| 0 | 12.0700 |
| 10 | 3.9000 |
| 20 | 1.4100 |
| 30 | 0.6120 |
| 40 | 0.2840 |
| 50 | 0.1420 |
| 60 | 0.0813 |
| 70 | 0.0506 |
| 80 | 0.0319 |
| 90 | 0.0213 |
| 100 | 0.0148 |

Table 6: Literature discrete values of glycerol viscosity

#### 3.7.1 Two Constant Equation

One equation used by scientists (Robert C. Reid, 1987) (DABIR S. VISWANATH, 2007) is of the form

$$\eta = AT^B$$

where $\eta$ is viscosity in $cP$, $T$ is the temperature in Kelvin and $A$ and $B$ are constants to be found.
The equation found by Robert C. Reid (Robert C. Reid, 1987) and others where $T$ is in Kelvin ($K$) and $\eta$ is in centipoises ($cP$) is the following:

$$\eta = (3.4263 \times 10^{73})T^{-28.52}$$

In particular the literature graph values are $A = 3.4263 \times 10^{73}$ and $B = -28.52$. To be able to compare, I have converted the discrete literature values and my values into Kelvin and centipoises. In particular, for some temperature $x$ in $^{\circ}C$, its temperature in Kelvin $K$ can be found using $x + 273.15$. Poises, on the other hand, is defined as $1P = 1gcm^{-1}s^{-1}$ whereas Pascal second is $1Pas = 1kgm^{-1}s^{-1}$. We know $1kg = 1000g$ and $1cm = 0.01m$, hence $1Pas = 10gcm^{-1}s^{-1} = 10P$. The converted values are found in the tables below and these values were plotted in a graph with best curves of fit to compare coefficients $A$ and $B$:

| **Temperature** $T/K$ | **Viscosity** $\eta/cP$ |
|---|---|
| 273.15 | 12070 |
| 283.15 | 3900 |
| 293.15 | 1410 |
| 303.15 | 612 |
| 313.15 | 284 |
| 323.15 | 142 |
| 333.15 | 81.3 |
| 343.15 | 50.6 |
| 353.15 | 31.9 |
| 363.15 | 21.3 |
| 373.15 | 14.8 |

Table 7: Literature values converted

| **Temperature** $T/K$ $\Delta T = 0.05$ | **Viscosity** $\eta/cP$ |
|---|---|
| 273.15 | $22200 \pm 1100$ |
| 278.15 | $10300 \pm 500$ |
| 283.15 | $5450 \pm 120$ |
| 288.15 | $5280 \pm 130$ |
| 293.15 | $4170 \pm 60$ |
| 298.15 | $3220 \pm 160$ |
| 303.15 | $2210 \pm 90$ |

Table 8: My values converted

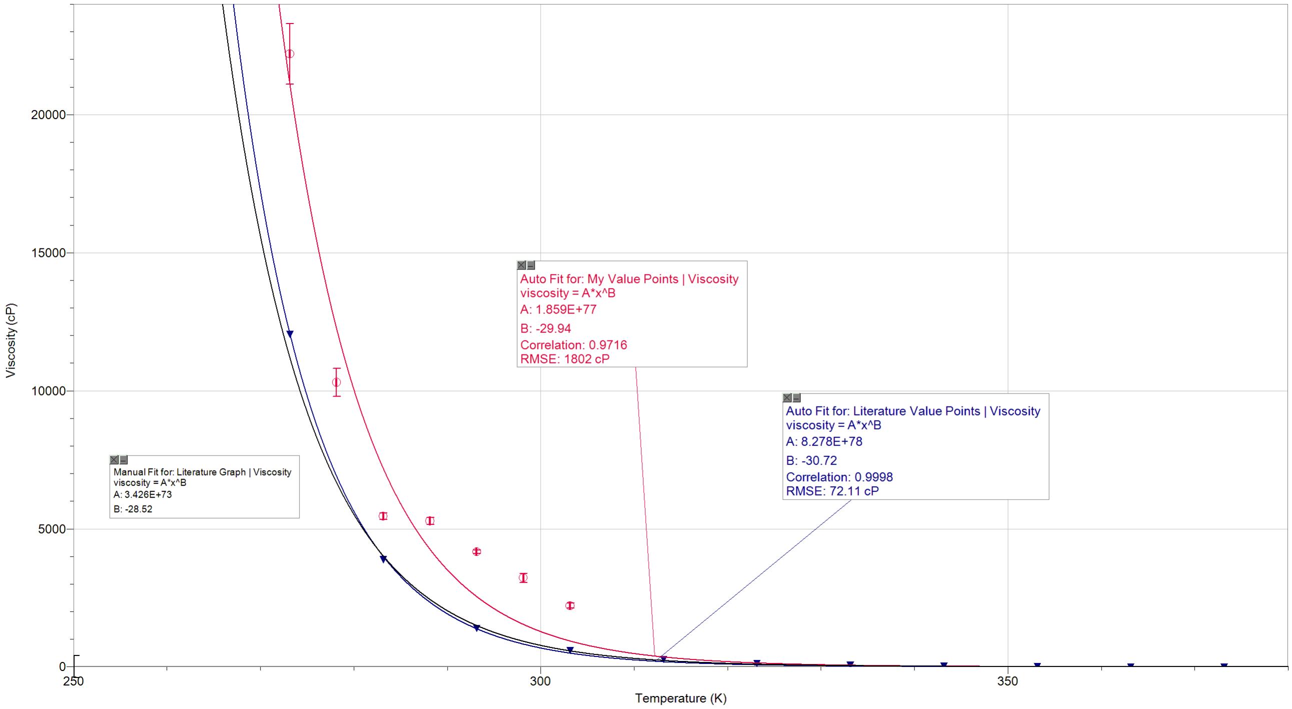


Figure 3: 3 Autofit curves and data sets for two constant equation

A great difference can be seen between my values and literature values. The red graph which represents a curve

of best fit using discrete literature values is very similar to that of the literature graph suggested in another source. The difference in the values of coefficients of their graphs could be explained by the fact that the graph generated by the discrete values (Association, 1963) is done taking into account values of $0, 10, \cdots, 100°C$, whereas the graph literature coefficients presented by Robert C. Reid takes into account values in the range $0 - 30°C$ inclusive. My discrete values for viscosity are significantly higher than literature discrete values. For example, at $273.15K$, the discrete literature value is $12070cP$ whereas my recorded value is $22200 \pm 1100cP$. This resulted in the graph of best fit to also increase at an earlier temperature, causing relatively large coefficients for $A$ and $B$.
Unfortunately, the graph of best fit for the two constant model has a very large root mean square error (RMSE) of $1802$, and does not pass through any of my points or error bars except the point at $T = 273.15K$. In comparison, the best fit curve for literature discrete values has a low RMSE of $72.11cP$ suggesting its accurateness and precision as the model is suitable for use in another scientist's literature values. This is further supported by the similarity between the graph generated using literature values and the graph coefficients found by another scientist.
The deviations of points from my curve of best fit are caused by random errors that occurred when during the experiment despite many precautions. Furthermore, as all values of my experiment go over consistently that of literature values, it is likely that a systematic error occurred too. However, this model is sufficient to show and supports the idea that there is an exponential increase in the values of viscosity, which can be seen from my individual values and high correlation values for both curves of best fit. In particular, where the number $1$ shows a strong correlation and is the highest number that one could theoretically achieve, my values and literature values achieved a correlation of $0.9716$ and $0.9998$ respectively. This implies that my values were still able to show a general exponential trend when compared with this model, despite the high presence of anomalies and errors.

### 3.7.2 Linearising The Two Constant Equation

In addition to comparing the curve of best fit, I have decided to linearise all discrete values to obtain linear graphs to conduct further analysis. We know that our model is $\eta = AT^B$, therefore we can logarithmate both sides:

$$\ln \eta = \ln AT^B \tag{4}$$
$$\implies \ln \eta = B \ln T + \ln A \tag{5}$$

which is comparable to our linear equation

$$y = mx + b$$

where $y = \ln \eta$, $x = \ln T$, $b = \ln A$ and $m = B$, therefore we must plot $\ln \eta$ against $\ln T$. To calculate the uncertainties, I will consider all min and max values from uncertainties for each individual discrete temperature and viscosity and linearise them. I will then find the uncertainty by considering the difference between min and max values and dividing it by 2:

| **Temperature Min** $T_{min}/K$ | **Temperature Max** $T_{max}/K$ | **Log Temperature Min** $\ln(T_{min})/K$ | **Log Temperature Max** $\ln(T_{max})/K$ | **Uncertainty of Temperature** $\Delta \ln(T)/K$ |
|---|---|---|---|---|
| 273.10 | 273.20 | 5.6098 | 5.6102 | 0.0002 |
| 278.10 | 278.20 | 5.6280 | 5.6283 | 0.0002 |
| 283.10 | 283.20 | 5.6458 | 5.6462 | 0.0002 |
| 288.10 | 288.10 | 5.6633 | 5.6637 | 0.0002 |
| 293.10 | 293.20 | 5.6805 | 5.6809 | 0.0002 |
| 298.10 | 298.20 | 5.6974 | 5.6978 | 0.0002 |
| 303.10 | 303.20 | 5.7141 | 5.7144 | 0.0002 |

Table 9: Linear temperature uncertainty

| **Temperature**<br>$\eta/cP$<br>$\Delta T = 0.05$ | **Viscosity Min**<br>$\eta_{min}/cP$ | **Viscosity Max**<br>$\eta_{max}/cP$ | **Log Viscosity Min**<br>$\ln(\eta_{min})/cP$ | **Log Viscosity Max**<br>$\ln(\eta_{max})/cP$ | **Uncertainty of Viscosity**<br>$\Delta \ln(\eta)/cP$ |
|---|---|---|---|---|---|
| 273.15 | 21100 | 23300 | 9.957 | 10.06 | 0.05 |
| 278.15 | 9800 | 10800 | 9.190 | 9.287 | 0.053 |
| 283.15 | 5330 | 5570 | 8.581 | 8.625 | 0.022 |
| 288.15 | 5150 | 5410 | 8.547 | 8.596 | 0.025 |
| 293.15 | 4110 | 4230 | 8.321 | 8.350 | 0.015 |
| 298.15 | 3060 | 3380 | 8.026 | 8.126 | 0.050 |
| 303.15 | 2120 | 2300 | 7.659 | 7.741 | 0.041 |

Table 10: Linear viscosity uncertainty

Thus my discrete literature values and my values are after logarithming and plotting in a graph are found below:

| **Temperature**<br>$\ln T/K$ | **Linearised Viscosity**<br>$\ln \eta/cP$ |
|---|---|
| 5.6100 | 9.398 |
| 5.6460 | 8.269 |
| 5.6807 | 7.251 |
| 5.7142 | 6.417 |
| 5.7467 | 5.649 |
| 5.7781 | 4.956 |
| 5.8086 | 4.398 |
| 5.8382 | 3.924 |
| 5.8669 | 3.463 |
| 5.8948 | 3.059 |
| 5.9220 | 2.695 |

Table 11: Literature linearised discrete values

| **Temperature**<br>$\ln T/K$<br>$\Delta \ln T = 0.0002$ | **Linearised Viscosity**<br>$\ln \eta/cP$ |
|---|---|
| 5.6100 | $10.008 \pm 0.05$ |
| 5.6282 | $9.239 \pm 0.053$ |
| 5.6460 | $8.603 \pm 0.022$ |
| 5.6635 | $8.572 \pm 0.025$ |
| 5.6807 | $8.336 \pm 0.015$ |
| 5.6976 | $8.077 \pm 0.050$ |
| 5.7142 | $7.701 \pm 0.041$ |

Table 12: My linearised discrete values

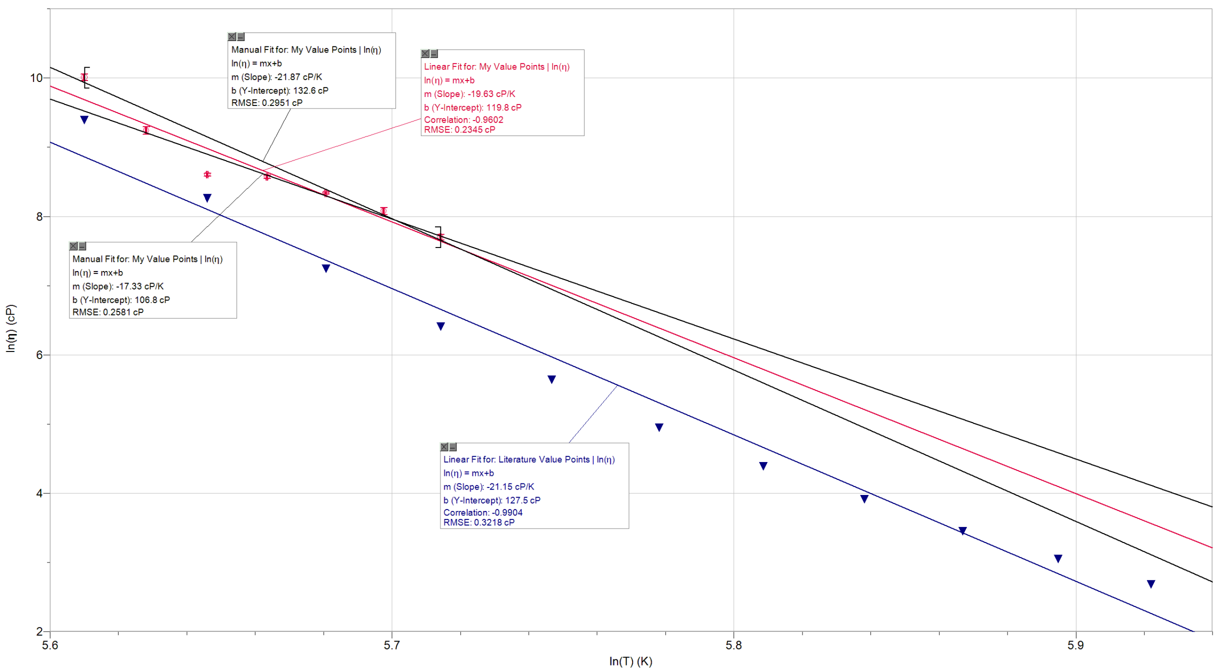


Figure 4: Linearised equations and lines of best and worst fit

Because the model I've used is only an empiricist equation which is used to describe an unknown relation, it could be seen that even after linearisation the points for my literature values are not perfectly linear and is slightly curved like an another exponential relationship, with the same applying to my own points.

The Pearson correlation test that of $-0.9602$ suggests that our line of best fit shows a very strong negative correlation as expected from my hypothesis. Furthermore, I attempted to draw the lines of worst fit by considering as many error bars and points as possible. For the lines of worst fit, the anomaly that was originally found at $T = 283.15K$ was ignored. Ideally from these lines, the gradient $m$ must fall between $-17.33 \leq m \leq -21.87$, for which indeed the literature value gradient is $-21.15$. Our line of the best fit's gradient for my points is $-19.63$, which is a $-7.2\%$ relative error. This means that despite having higher values for each of the points, the general change in viscosity with respect to temperature was captured well.

# 4 Conclusion

My question "How does the temperature affect the viscosity of glycerol?" with my hypothesis that lower temperatures of glycerol will result in higher viscosity and higher temperatures will result in lower viscosity seems to be supported by my data. Although a fixed equation is not known for the relationship between temperature and viscosity, one of the models that is widely used seems to well support a negative correlation when linearised as seen in Figure 4, in particular with my results that

$$\ln \eta = -19.63 \ln T + 119.8$$

with a strong correlation of $-0.9602$. When contrasted the linearised relationship with the full equation line of best fit, the linearised relationship allows for more a detailed analysis and makes this relationship stand out. Unfortunately, however, the line of best fit does not pass through all the points and their error bars due to a high prevalence of errors that occurred in the experiment.

My findings link back to my initial statistics of how the cold can negatively affect human health. Indeed, even though glycerol is a substance that is found in moderate amounts in the blood, it does show that a general decrease in temperature within a liquid will cause its viscosity to be higher and thus it will be harder to flow through the body. In order to compensate for this, the heart works at a faster rate which I believe can be the cause of heart attacks and more deaths in winter and colder temperatures. One thing, however, did not change: I will still require to wear at least 3 layers of clothing in the winter where I live.

# 5 Evaluation

## 5.1 Evaluation

A lot of heat was absorbed by the surroundings from the glycerol, otherwise, surrounding heat was also absorbed by the glycerol. This made it especially difficult to ensure that the data is for the temperature intervals as planned causing systematic errors. Although precautions were taken such as surrounding the measuring cylinder with heat-insulating materials, in order to minimise energy transfer, a more accurate way to achieve desired temperatures would be to in fact take into account Newton's law of cooling equation in order to predict what initial temperature the glycerol has to be poured in order to make measurements more accurate depending on time $t$ after pouring.

One source of random error that I have noticed during the experiment is that during heating and cooling and pouring, air can be entrapped inside glycerol forming bubbles which in result decreased the viscosity as the resistance of air on a ball is minimal in comparison to that of glycerol. This would explain the value of viscosity found at $T = 283.15K$. One way of avoiding this is by placing the liquid in a vacuum to ensure that all air is sucked out.

Furthermore, the great difference in the viscosity is also staggering, as my recorded points are well over when contrasted the viscosities at temperature values of $0, 10, 20$ and $30°C$. The consistency of this difference suggests a systematic error. A comparison of data for the density of glycerol using literature values from (Association, 1963) suggests that densities are measured to sufficient accuracy, implying that it is not the cause of the error. This leaves the measured velocity as the culprit for this uncertainty. In order to reduce this systematic error, it is very much possible that the velocity is affected by the fact that the sphere is big in comparison to the hole it travels through. This hinders the effect of Stokes' Law, as it is assumed that there is enough space for the particles to be pushed away in a laminar manner, where the streamlines dissipate and straighten as they move further away from the sphere. Unfortunately, it is likely that the lack of space did not allow a laminar stream, meaning that the fluid was acting more against the sphere as a result of the disturbance of streamlines causing turbulence. In order to improve the proportions of free space in comparison to the sphere size, one way to reduce the error would be to have a smaller sphere to drop through, or find a measuring cylinder with a larger radius. Utilising both effectively would likely minimise my systematic error for the velocity. Moreover, another change that can aid reduce the error for my velocity is to introduce light gates. The limitation of using a camera to measure the time taken is that it is limited to its frames per second. In comparison, the iPhone camera was only 60 FPS, meaning that it skips small intervals of time in slow motion as I was analysing. The use of light gates to measure the time to pass between two points of light would make it more accurate as such limitations are very minimal when utilising the speed of light.

Another limitation is that the model used for this internal assessment is not necessarily correct in explaining the behaviour of viscosity in relation to temperature. Since the relationship is described as very complex, one way to come closer to this complexity is to use models which can explain the behaviour more accurately by considering more constants. For example, one such model (Robert C. Reid, 1987) is

$$\ln \eta = A + \frac{B}{T} + \frac{C}{T^2}$$

where $\eta$ is viscosity in $cP$, $T$ is temperature in $K$ and $A, B, C$ are constants to be found.

This, however, would not describe the behaviour perfectly until a formulaic relationship between temperature and viscosity is found. This limitation explains why even the literature values in Figure 4 are not perfectly linear where even the best fit line for literature values does not pass or go close to all points. Nevertheless, this is currently the best method that we can utilise to achieve and interpret results.

Despite the high presence of systematic and random errors which affected the accuracy and precision of my results, after linearising a strong negative correlation is still evident implying the validity of my hypothesis.

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